

Problem Solving Through Estimation

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Abstract: Human being is the most evolved form of life on this planet, with highest level of cerebral capacity which means a human being can think, remember, have memory of vivid sense and fantastic sense of imagination. This species is armed efficiently, to solve any type of problem it confronts. Its immense potentiality guides it to adopt the best approach or strategy to solve any problem. In this direction, the notion of developing children's problem solving and reasoning skills has been embraced. To work upon problems, estimation is an effective and valuable approach. Estimation is an educated guess based on richness of memory, present experience and higher level of imagination. Estimation as a process involves perception and conception of the problem, relate it to information that is already known, judge and verify reasonableness, and revise as necessary for the most appropriate solution of the problem.

Key Words: *Problem Solving, Problem, Estimation, Memory and Experience*

Date of Submission: 18-05-2019

Date of acceptance: 03-06-2019

I. INTRODUCTION

Important component of problem-solving skills is estimation where one has to learn the scales and dimensions of the problem for appropriate action in that environment. Estimation activity results in students to become actively involved in the exploration of spatial thinking in the process of solving the problem. Problem-solving as described by the NCTM (2000)¹: Problem-solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must focus on their skills and strategies, while processing the problem, hence they will often develop new mathematical understanding.

Problem-solving is an aspect of mathematics that encompasses thinking, communication, and understanding. Sproull and Hofmeister (1986)² suggest that an individual's response to a new idea or problem involves cognitive processes of interpretation, attribution, and inference. A brief description of these processes is provided below: 1. Interpretation - This process involves the formation and transformation of a mental picture or model of a problem or situation. This occurs as the individual visualizes and scans the problem, relevant information, and the behavioural or attitudinal changes that are needed to integrate an idea or to solve a problem. 2. Attribution - This process involves an assessment of the probable causes of difficulty that exist for the implementation of a new idea or problem solution. Environmental scanning occurs as the individual collects relevant data for this assessment. 3. Inference - This process involves estimation of the usefulness of an idea or solution and a determination of the degree of ease or difficulty for implementation of an idea or problem solution.

The study of student's estimation reflects a direction of their thinking and problem solving. An estimate as referred to in the National Curriculum involves checking that calculations and measurements are reasonable (DES 1991b)³. It involves the process of accumulation and linking of prior experience, memory, and imagination in an attempt to determine a method for resolving a situation whose outcome is not known. An everyday working knowledge of any measurement system depends on the ability to estimate reasonably rather than the ability to convert units precisely. Conceptualizations of the development of children's quantitative skills treat estimation as a microprocess, or "operator" (Gelman, 1972⁴; Gelman & Gallistel, 1978⁵; Klahr & Wallace, 1976⁶; Piaget, 1952⁷; Piaget, Inhelder, & Szeminska, 1960⁸). The alternative view of estimation as a macroprocess has received little consideration within the contemporary literature on quantitative skills.

Mental image formation of different dimensions of the problem in relation to its environment and its transformation as per the requirement plays a vital role in problem solving through estimation. Integration of a new

knowledge to the existing ones occurs. Estimation involves the assimilation of new knowledge and adaptation of the existing ones with respect to the new in the cognitive structure of mind.

It is possible, however, to conceive of estimation as a higher-level strategy or problem-solving process that can be superimposed on and integrate subitizing operators, or that can be invoked when counting is expensive or inefficient.

II. MEANING OF PROBLEM IN PROBLEM SOLVING

A problem can be defined in several ways depending on the person. Some may think that figuring out a change in one's behavior in different phases of life or figuring out what one is going to do the next day is solving a problem. All have problems, but what may be a problem for one, may not be for the next person.

Therefore, a task that may be assigned to a student may or may not be a problem. Charles and Lester (1982)⁹ divide the definition of a problem used for problem-solving into three components. The first is "the person confronting it wants or needs to find a solution". This requires the teacher to present a problem that the student will want to solve. The NCTM (2000)¹ suggests that the teacher provide problems that arise in mathematics and other contexts, and are of interest to students. The second is "the person has no readily available procedure for finding the solution", meaning that the student cannot recall a direct method of solution or an algorithm to use in solving the problem. The third is "the person must make an attempt to find a solution". There must be some sort of effort made to solve the problem in order for the student to become a successful problem solver. Thus, the teacher must take into account the differences in knowledge, experience, ability, and interests of the students (Charles & Lester, 1982)⁹.

Six types of problems. Charles and Lester (1982)⁹ also describe six different types of problems used in a mathematics curriculum, not necessarily for problem-solving. The first is drill exercises, which emphasize basic computational skills such as $641 \times 22 = 14102$. The second type of problem is simple translation, where the solution can be translated into a simple expression.

The third is a two steps translation problem called complex translation. The fourth are process problems, which require problem-solving methods such as understanding, planning, solving, and attempting a solution. Charles and Lester (1982)⁴ suggest using the following problem-solving strategies, developed by Polya (1957)¹⁰ • Look for a pattern. • Guess and check. • Draw a picture. • Make an organized list. • Make a table. • Work backwards. • Use objects or act out the problem. • Use logical reasoning.

The fifth type of problem is an applied problem. An applied problem uses mathematics in a realistic type of situation. Students find the solution using mathematical skills, facts, concepts, and procedures as well as organization and summarizing skills. An applied problem sometimes requires skills to construct graphs, computation, and estimation.

The final type of problem is a puzzle problem. The methods used to solve these types of problems include either guess and check or thinking in a way that is not obvious to anyone else. There are usually tricks involved in these types of problems where the student either likes that type of frustration or else they quite early. Hence, to solve any type of problem an appropriate process is must. Problem solving involves different strategies. Selection of the best strategy and its verification brings up the reasonable estimate.

III. PROBLEM SOLVING PROCESS

The NCTM (2000)¹ state "the essence of problem-solving is knowing what to do when confronted with unfamiliar problems". Gagne (1980)¹¹ emphasizes the criticality of problem-solving to scientists and practitioners in many disciplines, while stating that problem-solving is a primary factor in and consequence of learning. An individual gains new knowledge through the articulation of and expansion of his/her cognitive structure. This enhanced cognitive structure then serves as the foundation for information search and processing as well as in problem-solving (Gagne, 1977¹²; Gagne and White, 1978¹³).

Polya (1957)¹⁰ created the four-step approach to problem-solving in order for students to be able to learn 'how to think,' not 'what to think' or be told 'what to do'. In 1957, he wrote the book, *How to Solve It*, defining four specific steps to problem-solving.

The first step is 'Understanding the Problem.' When the student is trying to solve the problem, he or she must first think about, "What is the unknown? What is the data? What is the condition?". At this stage, the student should want to solve the problem and understand it well enough so that they are able to list the main points, the unknown, the data, and what the question is asking (Polya, 1957)¹⁰.

Once understanding the problem is established, the student can move onto step two, which is 'Devising the Plan.' In 'Devising the Plan,' a connection should first be made between 'What is the data?' and 'What is the unknown?' Once this is achieved, then the student should be thinking about the following questions: "Have you seen

the problem before? Or in slightly different form?”. If the student has seen the problem before or one similar, then they should be able to think about how the problem was solved previously. If the problem cannot be solved readily, then the questions to think about are: “Could you solve a part of the problem? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown?”. In order to help students with ‘Devising a Plan,’ Polya (1957)¹⁰ developed problem-solving strategies such as ‘Look for a Pattern’ or ‘Guess and Check’ as listed above in the definition of a problem (Charles & Lester, 1982)⁹.

The third step is ‘Carrying Out the Plan.’ The questions to think about are: “Can you see clearly that the step is correct? Can you prove that it is correct?”. The plan is an outline of the strategy chosen and what gives the student a direction to solving the problem.

Finally, the fourth step is ‘Looking Back.’ In this step, students are checking their solution for computational errors as well as reflecting on whether their solution is sensible and then communicating the process, they used to solve the problem. In this step, advises students to close their books, think and reflect on the following questions: ‘Does the solution seem reasonable? Did I answer the question asked? Was the plan carried out correctly?’

Hunt (1982)¹⁴ emphasizes the importance of problem-solving to other mental processes that allow individuals to recognize contrasts and similarities in their worlds. When the problem-solver makes the problem simpler, one aspect of this to guess which parts are vital elements of the problem and which can be ignored. Estimating, as defined, may be an important tool in the problem-solver’s repertoire.

IV. MEANING OF ESTIMATION

Estimation is a process that starts with a problem in the real world and ends with an inexact quantitative statement. Although estimation is guessing, it is educated guessing. Bright (1979b)¹⁵, stated that: “Understanding estimation is important for understanding measurement. Every measurement is an approximation, or if you will, an estimate”. Instead, estimating requires that a judgment be made about an object’s approximate relationship to a standard. While estimation one observes the properties of object, order in terms of size and sequence, analyse the creativity of designs. For example, as to reason for football to be hollow, comparing the similarities and differences of object with others. Two major themes of the NCTM’s Curriculum and Evaluation Standards for School Mathematics (1989)¹⁶ are embedded in the process of estimation: (a) connections through linking mathematical ideas to the physical world and (b) communication through articulation of ideas.

In this context St. John¹⁷ argues that instruction in estimation is critical to skilled measurement in that it can be done quickly, can serve as a check on measurements (and can often replace them), and can reinforce understanding of the quantity being measured by causing one to rehearse the measurement procedure mentally. St. John’s work with real-world estimation rests on implicit principles about how estimation problems are represented cognitively and about how estimation skill can be facilitated. NCTM Standards document (1989)¹⁶ states that, “Estimation interacts with number sense and spatial sense to help children develop insights into concepts and procedures, flexibility in working with numbers and measurements, and an awareness of reasonable results”. An individual’s cognitive processing style is posited to affect the manner in which information is represented or processed during learning and thinking. Estimation is the strictly mental process of arriving at a measurement without the aid of measurement instruments or tools. Every measurement is, after all, an estimate, with the measurer estimating to the nearest unit.

One tries to fix a problem through vivid information, imagination and experience. Role of estimation becomes imperative to check the usefulness of a solution as well as the difficulty or ease with which it can be implied.

V. SIGNIFICANCE OF ESTIMATION IN PROBLEM SOLVING

A person confronts a problem, has no readily available procedure to find the solution. It is his vivid imagination that makes him to think in multi direction based on his memory and prior experience. A person ponders upon different ideas or solution. One should look pass all the additional information that is irrelevant and get straight into what is important and relevant, should not be distracted by other irrelevant information.

Usefulness of the solution makes it best option. It is estimation which makes one opt for the most useful one. Persons’ commitment towards estimation is an important factor for most appropriate solution. Similarly, very important is the fact that a person confronting a problem wants to find a solution and hence, is ready to make an attempt or not.

In different type of problems as mentioned by Charles & Lister (1982)⁹, applied problem and puzzled problem mainly involves skill of estimation. Applied problem mainly involves use of mathematics in realistic life

situation, but however, in such cases it is not always possible to have some mathematical formula or measurement unit and tool to solve problem, estimation then serves the purpose. Puzzled problems include guess and check method. Estimation is after all an educated guess. Estimation gives an opportunity to explore different strategies and use the most suitable one. The solution is to be verified and checked as it must be reasonable and useful.

One's thinking makes one generate numerous alternatives to solve a problem, estimation makes out the best among those alternatives after verification of the solution. Generation of alternatives refers to brainstorming an exhaustive list of possible solutions appropriate to the problem. Estimation refers to determine the utility and effectiveness of all solutions, the consequences of the solutions, and identifying the optimal solution i.e. most reasonable.

Estimation also give scope for modification of the solution by change in the strategy opted if the solution is not found to be most appropriate. Verification refers to the assessment of the actual outcome of the chosen solution after the course of action has been taken, to possibly correct or to modify actions.

Benefit of estimation in problem solving is that it makes easier for a person to select best from its varied strategies. It gives opportunity to compare numerous and select the most appropriate. In one way it is better approach as is different from the approach with single process and single solution. It also develops thinking power of a person in different directions and figure out if different ideas are relevant to the problem. It somewhere widens the horizon of thinking as one step into imagination to find a solution.

Filtration of the strategies and verification which sometimes leads to modification of the strategy brings to the surface the best and most suitable. So, estimation in no way leave any corner of a problem that is not scanned, of an environment that is not noted and its verification process proves it to make the strategy as well as the solution the most valid one.

VI. PROBLEM SOLVING THROUGH ESTIMATION

The great problem-solver, G.Polya (1981)¹⁰, described the scientific method as "Guess and Test" and suggested that teachers should encourage pupils to guess with the exhortation. Estimation is an educated guess. The literature of mathematics education has argued repeatedly that measurement is most successfully taught when estimating is one kind of instructional activity (Bright, 1976¹⁸; NCTM, 1989¹⁶; O'Daffer, 1979¹⁹; Rubenstein, 1985²⁰). Role of instruction is imperative in estimation. Estimation starts with a perception of what is to be estimated. What is the dimension of the problem which is to be estimated? Students begin by perceiving the properties of objects through their different senses vision (sight), gustation (taste), audition (hearing), olfaction (smell) and somato-sensation (touch). More the sensory organs involve in perception of the problem better is the estimation.

Processing of the stimuli takes place in mind where they eventually begin to internalize that some attributes of objects are measurable (i.e. the length of their hand), while others are not (i.e. the hardness or softness of their hand). Having perceived a property of an object, students get the details of data which is made available to them in form of properties. What are the known features of the problem based on prior knowledge? Is the existing problem exactly the same as that of confronted in past? Students try to find a suitable referent benchmark depending on the prior experience. If the student can find some benchmark exactly same then it can be applied readily. How was the previous problem solved? Once the student is able to recall the approach, a progress can be made. Or if slightly different? Then must think of some other benchmarks that would suit the problem through some modifications. For example, use of that referent benchmark for multiple times in case it is small or to take fraction of the benchmark in case it is large. If still not found, then they naturally want to compare it with other objects having the same property. Comparisons of sensations are quite natural consequences of perceptions. Comparing two objects is adequate when one wish to make gross statements of equivalence or non-equivalence, such as Rahul is taller than Karen is. However, as soon as it becomes apparent that this approach of comparison is quite ineffective. Other strategies can be worked upon such as recomposition or decomposition of the problem where atleast a part of the problem should be tried to solve by regularizing a problem by assuming regularity and then averaging. The result of estimation must seem to fit and be reasonable to serve the purpose. Unless and until one find a reasonable estimate, one should work with some better modifications to be more accurate in their estimation (Siegel, Goldsmith & Madson, 1982)²¹

Students with good estimation skill have multiple strategies available to them. Thus, they are not only more likely to have a repertoire of strategies available to them, but they are also more likely to choose the strategy most appropriate to the situation. Selection of best strategy assures estimation towards perfection. In order to find reasonable estimate, the task to be estimated is based on clear understanding of the problem. The problem itself should help the estimator in forming mental image and hence in processing of best suited strategies in the mind. The

factors contributing to the dimension of the task to be estimated must be taken into account. The intervening variables must be overcome. Simple previous problem is used as a base to solve complex ones. An individual widens the horizon of their existing cognitive structure through accommodation and assimilation of new ideas to the existing ones. This enhanced cognitive structure then serves as the foundation for problem-solving. Complex cognitive structure provides the opportunity to apply previous learnings to existent problems and to integrate new learnings that are relevant to future problems.

There is significant role of environment on estimation. Hence, different environmental factors which has direct effect on the result must be taken into concern. One must be sure and conscious enough to select a proper strategy and follow it to solve the problem and also make certain required modifications in it.

VII. CONCLUSION

Problem-solving is an aspect of mathematics that incorporates thinking, understanding and communication. Problem solving includes the instructions for the process and description of the problem, the problem-solver's previous experience of the problem and the new knowledge one constructs while problem solving. Estimation is the skill used in problem solving where students understands the basic nature of problem and thus devise a plan to counter it.

Different types of problems exist where applied problems and puzzled problem usually involves estimation. Estimation involves higher level thinking in problem solving that depends on the ability to operate symbols in a comprehensible manner so that problems are first conceptualized in a meaningful context and then managed effectively. Selection of most appropriate strategy is imperative for perfect estimation, where the problem-solver makes the problem simpler, one aspect of this to guess which parts are vital elements of the problem and which can be ignored. This allows application of prior knowledge and permits modifications if needed. This whole process demands a solution to the problem through estimation. Reasonable estimate justifies the strategy used in problem solving.

Hardly, any attention as to how a problem is solved? A human being lives because of richness of his memory, the present experience and vividness of imagination for future is important. All gotten mixed up, because the nature of subject has not come. Real understanding and feel of the subject come when one gets in touch with the nature of the subject. Estimation brings into effect the basic understanding of the nature of subject. Hence, estimation counters problem effectively.

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IOSR Journal Of Humanities And Social Science (IOSR-JHSS) is UGC approved Journal with Sl. No. 5070, Journal no. 49323.

Rinki Tiwari. "Problem Solving Through Estimation." *IOSR Journal of Humanities and Social Science (IOSR-JHSS)*. vol. 24 no. 06, 2019, pp. 01-06.